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Strong Transverse Coupling in the Tevatron

Experiments and analyses by G. Annala, J. Carson, D. Edwards, N. Gelfand, D. Harding, T. Johnson, J. Johnstone, M. Martens, T. Sen, M. Syphers

Reported by Don Edwards and Mike Syphers

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1 Introduction

The Tevatron was designed with an extensive set of correction and adjustment magnets built into the spool pieces in recognition of the circumstance that a superconducting synchrotron was not as easy to modify as its conventional forebearers. Recently, concern has mounted at the high excitation of the skew quadrupole correctors. The purpose of this note is to account for this situation.

When slow extraction was attempted from the Main Ring in the summer of 1970 horizontal-vertical coupling prevented adequate transverse oscillation growth for efficient slow spill. This situation was corrected by an 8 mrad roll of each of twelve equi-spaced quadrupoles[1]. In order to avoid a repetition of this problem in the Tevatron, an extremely strong skew quadrupole circuit was built in at the outset. When the Tevatron was commissioned only 4% of the capability of this circuit was required. Now, 20 years later, the excitation of this skew quadrupole circuit is approximately 60%.

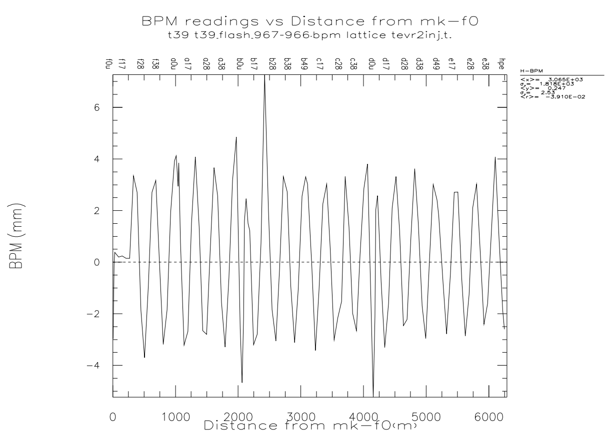
Other skew quadrupole correctors were installed in the neighborhood of the long straight sections, and for a variety of reasons the number of elements in the strong circuit was reduced from 48 to 42. These are relatively minor changes in the present context.

Recall that in the normal Tevatron tuning process the skew quad circuits are adjusted to minimize the difference between the horizontal and vertical tunes to the level of $\Delta\nu_{min} \approx 0.003$. Normally the horizontal-vertical coupling is not observed directly by orbit measurements during this procedure. It was recognized that the strength of the skew quadrupole settings would imply an uncorrected minimum tune difference of 0.2 units! Clearly, with the skew quad circuit turned off the coupling of the orbital motion should be easily observable.

In the following sections, we describe the recent Tevatron studies that exhibit the transverse coupling and the analyses that link these observations to the long term development of a skew quadrupole coefficient in the Tevatron dipoles. In brief, our conclusion is that a_1 at the level of one of our traditional units will account for the coupling and is consistent with physical examination of a selection of dipoles in the tunnel.

2 The Experiments of February 18 and 27, 2003

The goal of these study periods conducted by G. Annala was to look for the sources of transverse coupling by turning off the skew quadrupole correctors and injecting with a transverse offset in one degree of freedom and looking for growth in the other degree of freedom. As usual one makes use of orbit difference measurements. In Figure 1 the upper trace exhibits the progress of a horizontal oscillation throughout one turn in the Tevatron where the difference is caused by a steering dipole at F13. Note the progressive growth of the vertical amplitude in the lower trace. This was the first data to suggest a systematic skew quadrupole term in the ring. Corresponding data using a steering element one cell downstream (60°) was consistent. Attempts to find a significant localized disturbance by using a variety of steering dipole locations were not successful.



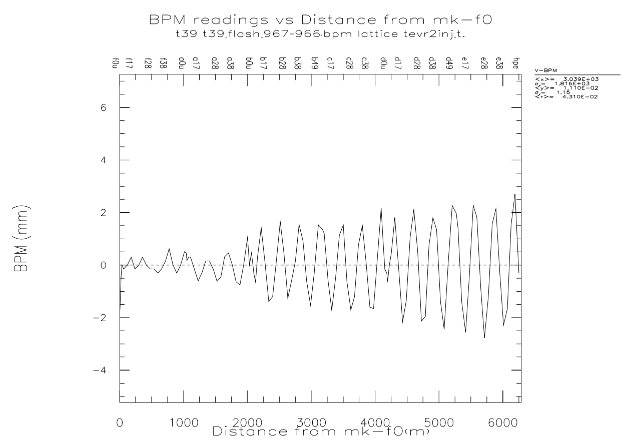


Figure 1: First-turn flash data taken on February 18, 2003 by G. Annala.

The measurements of both study periods are elegantly characterized by Figure 2 which shows the progress through several turns. This figure shows a difference orbit with an initial oscillation in the horizontal degree of freedom generated by a mistuned steering dipole in the injection transfer line. In textbook fashion within 1.5 turns the motion couples fully into the vertical and in another 1.5 turns returns fully to the horizontal.

Some pictures and data summary may be found in the Tevatron E-log entry for Thursday, February 27, 2003, 07:08.

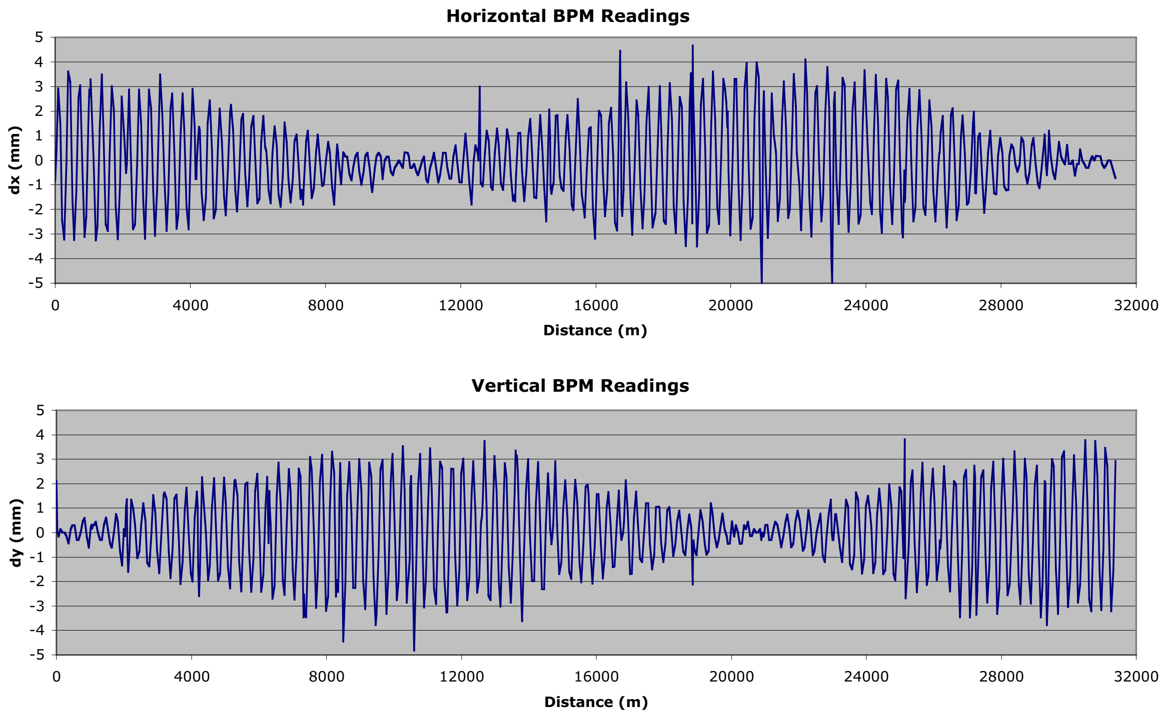


Figure 2: Flash data taken on February 22, 2003 by G. Annala. Here, data from 5 consecutive turns (courtesy N. Gelfand) have been concatenated to form this figure.

3 Analysis

Linear transverse coupling is commonly attributed to rolled quadrupole elements. Several attempts have been made by T. Sen, B. Erdelyi, M. Martens, and others to determine strong local sources of coupling in the Tevatron without much success. In terms of the roll angle ϕ of a single lattice quadrupole, minimum tune split is $\Delta\nu_{min} = 2\phi\sqrt{\beta_x\beta_y}/(2\pi F) \approx 2\phi/\pi$, where F is the focal length of the quadrupole. A single main quadrupole in the Tevatron generating a tune split of 0.2 units would imply a roll angle of 314 mrad (18°)! Indeed, the roll angles of almost all quadrupole and dipole magnets in the Tevatron have recently been measured and the resulting data can account only for a tune split an order of magnitude lower than what is observed.

The data of February 18, 2003 suggest that a systematic skew quadrupole component exists in the Tevatron distributed along the circumference. If the 200 quadrupoles were all rolled systematically, the roll angle would have to be 314 mrad / 200 $\approx$ 1.5 mrad, but this implies each focusing quad is rolled inward and each defocusing quad is rolled outward, for example. We also know from the January roll measurements that this is not the case. However, if all the dipole magnets had systematically developed a skew quadrupole component to their magnetic field, this could account for the observations. The skew quadrupole multipole coefficient, a_1, is defined by $a_1 = (\partial B_x/\partial x)/B_0$ where B_0 is the main dipole field strength. The minimum tune split due to a systematic a_1 in the Tevatron dipoles would be

$$\Delta\nu_{min} = \frac{1}{2\pi}\frac{B_0 a_1 \ell}{(B\rho)}\sqrt{\beta_x\beta_y}\ N_{dip} \quad (1)$$

$$\approx \frac{1}{2\pi} a_1\theta_0(2F)N_{dip} \quad (2)$$

$$= 2Fa_1 \quad (3)$$

and so a tune split of 0.2 would imply a value of $a_1 \approx 0.004/\text{m} = 1\times10^{-4}/\text{in}$ (1 "unit" of a_1, in the standard Fermilab Tevatron magnet system of units). Suspicion of a systematic a_1 in Tevatron dipoles had already been raised by measurements performed in the tunnel during the January 2003 shutdown period as reported by J. Carson and D. Harding. Physical measurements of Smart Bolt movement suggested an a_1 at the level of one unit. In this climate, it was natural that analysis of the beam measurements focus on this source of skew coupling.

3.1 Analytical Treatments

3.1.1 Reprise of 1970 Estimate

This is just a repeat of the 1970 calculation, with suspicion resting on the dipoles on this occasion. Localize the four dipoles between each pair of quadrupoles at the midpoint of the inter-quadrupole space. These four dipoles will represent a skew lens of focal length f given by $1/f = 4\theta a_1$ where θ is the 8 mrad bend of each dipole. Suppose a horizontal oscillation exists such that at the nth inter-quadrupole position the displacement is $x_n = x_0\cos(n\mu)$, where μ is the half-cell phase advance. At this location, a vertical oscillation will be initiated with deflection angle x_n/f. Downstream after N half-cells, the total vertical displacement may be approximated by

$$y_N \approx x_0 4\theta a_1\beta\sum_{n=0}^{N}\cos(n\mu)\sin[(N-n)\mu] \quad (4)$$

where β is the amplitude function midway between the quadrupoles. Ignoring the oscillatory terms in the sum, its amplitude is $N/2$.

The condition that the horizontal oscillation fully couple into the vertical plane is

$$x_0 4\theta a_1\beta N/2 = x_0, \quad (5)$$

from which

$$a_1 \approx 1.6\times10^{-4}\ \text{per inch} \quad (6)$$

where we have taken $N = 200$ to represent one turn and $\beta = 50$m.

3.1.2 Difference Resonance Analysis

In response to a question from M. Martens, T. Sen performed a calculation to confirm that it was possible for an oscillation in one transverse degree-of-freedom to couple fully into the other as illustrated in Figure 2 under present conditions.

Sen carries out his analysis in the canonical formalism, beginning with the Hamiltonian

$$\mathcal{H} = \frac{1}{2}\left[x'^2 + K_x x^2 + y'^2 + K_y y^2\right] + \frac{a_1}{\rho} xy \tag{7}$$

where the bracketed terms characterize the linear uncoupled motion. The last term represents the transverse coupling with the definition

$$a_1 \equiv \frac{1}{2B}\left(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\right) \tag{8}$$

With the reasonable assumption that the coupling is dominated by the $\nu_x - \nu_y$ difference resonance, Sen concludes that the behavior of Figure 2 will be exhibited by a sufficiently strong resonance driving term. The amplitude of this resonance driving term is the minimum tune split and is found to be $2091 \times a_1^M$ where $a_1^M = a_1/r_{ref}$. Thus a value $a_1^M = 1 \times 10^{-4}$ would imply a minimum tune split of 0.209, in agreement with the value obtained earlier. The phase of this resonance driving term is found to be -13 degrees and is about 3 degrees out of phase with the driving term due to the main T:SQ family of skew quadrupoles. Using only this family of skew quadrupoles to correct the coupling from the dipoles leaves a minimum tune split of 0.02. Combining the T:SQ with another family T:SQA0 of skew quadrupoles which are 38 degrees out of phase with the T:SQ family, the minimum tune split can be corrected to below 0.001. The analytically calculated strengths of these skew quadrupoles are -2.11 Amps in T:SQ and +2.32 Amps in T:SQA0 at 150 GeV.

The other main result in his note is that when the coupling is weak, the out of plane amplitude following a kick in one plane is directly proportional to the minimum tune split. The complete calculation may be found in [2].

3.2 FODO Matrix Calculation

In addition to the analytical estimates made above, the problem can be analyzed using standard 4×4 matrices of FODO cells with a skew quadrupole located in the middle of each half cell. If we take $k \equiv 4\theta_0 a_1$ as the equivalent skew quadrupole strength due to the 4 dipole magnets within a half cell (each with bend angle $\theta_0 = 8$ mrad), then the matrix through passage of one cell will be given by

$$\begin{aligned} M_{cell} \;=\; & \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/F & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/F & 1 \end{pmatrix} \begin{pmatrix} 1 & L/2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L/2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k & 0 \\ 0 & 0 & 1 & 0 \\ -k & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L/2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L/2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ & \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/F & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/F & 1 \end{pmatrix} \begin{pmatrix} 1 & L/2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L/2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k & 0 \\ 0 & 0 & 1 & 0 \\ -k & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L/2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L/2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

Passage through one turn of the Tevatron is roughly equivalent to the repeated application of the matrix M_{cell} about 100 times. For a simple model of the Tevatron, we take $L = 30$ m, $F = 25$ m, and look at $M_{ring} = M_{cell}^n$. For $n = 100$, and $k = 0$ (i.e., an uncoupled lattice), the base tunes are $\nu_x = \nu_y = 20.48$. When we take $k = 4\theta_0 a_1$, with $a_1 = 1$ unit, we find the eigentunes are $\nu_1 = 20.38$ and $\nu_2 = 20.58$, for which $\Delta\nu_{min} = 0.20$. If we start with an initially horizontal betatron oscillation with amplitude 4 mm, the motion couples into the vertical as seen in Figure 3, which looks strikingly similar to the February 18 data. Extending the plot for $n = 400$ (4 turns), we obtain Figure 4, in which we see the amplitude of the motion exchange from horizontal to vertical and back.

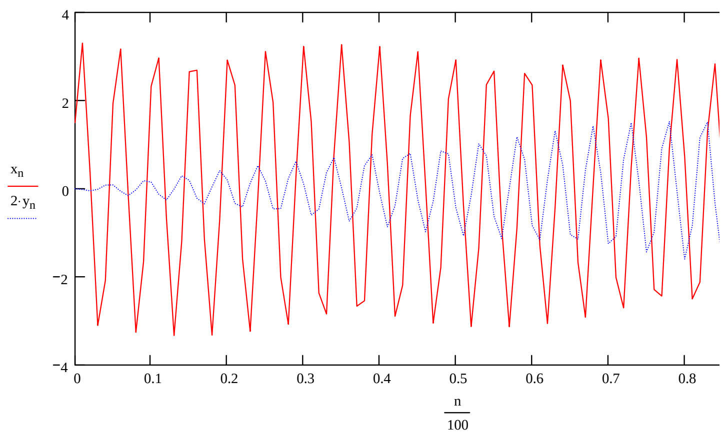


Figure 3: FODO calculation - 1 turn

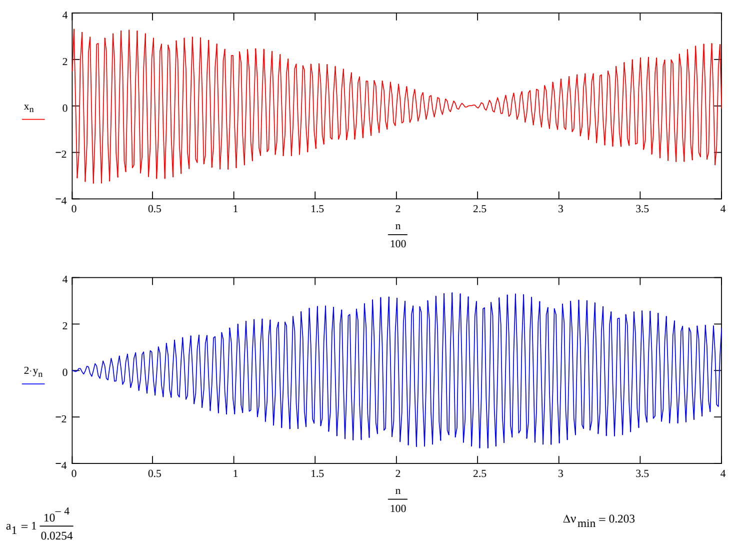


Figure 4: FODO calculation - 4 turns

3.3 Tracking Results

Unfortunately, for such a claim of understanding to be accepted by the most skeptical of critics one must perform computer simulations using sophisticated models of the Tevatron lattice. Two independent approaches have been undertaken, one by J. Johnstone using MAD and the other by N. Gelfand using TEVLAT. Figure 5 is the result of a MAD calculation in which each of the dipole magnets in the Tevatron was given a value of $a_1 = 1$ unit. No other magnet errors were included. As can be seen, the behavior seen in the data is well reproduced. Likewise, the TEVLAT results, one instance of which is shown in Figure 6, verify our understanding as well. In this figure, the model of the Tevatron includes all measured magnetic multipoles from the magnet database, with 1 unit added to the database values of a_1 for each of the Tevatron dipole magnets.

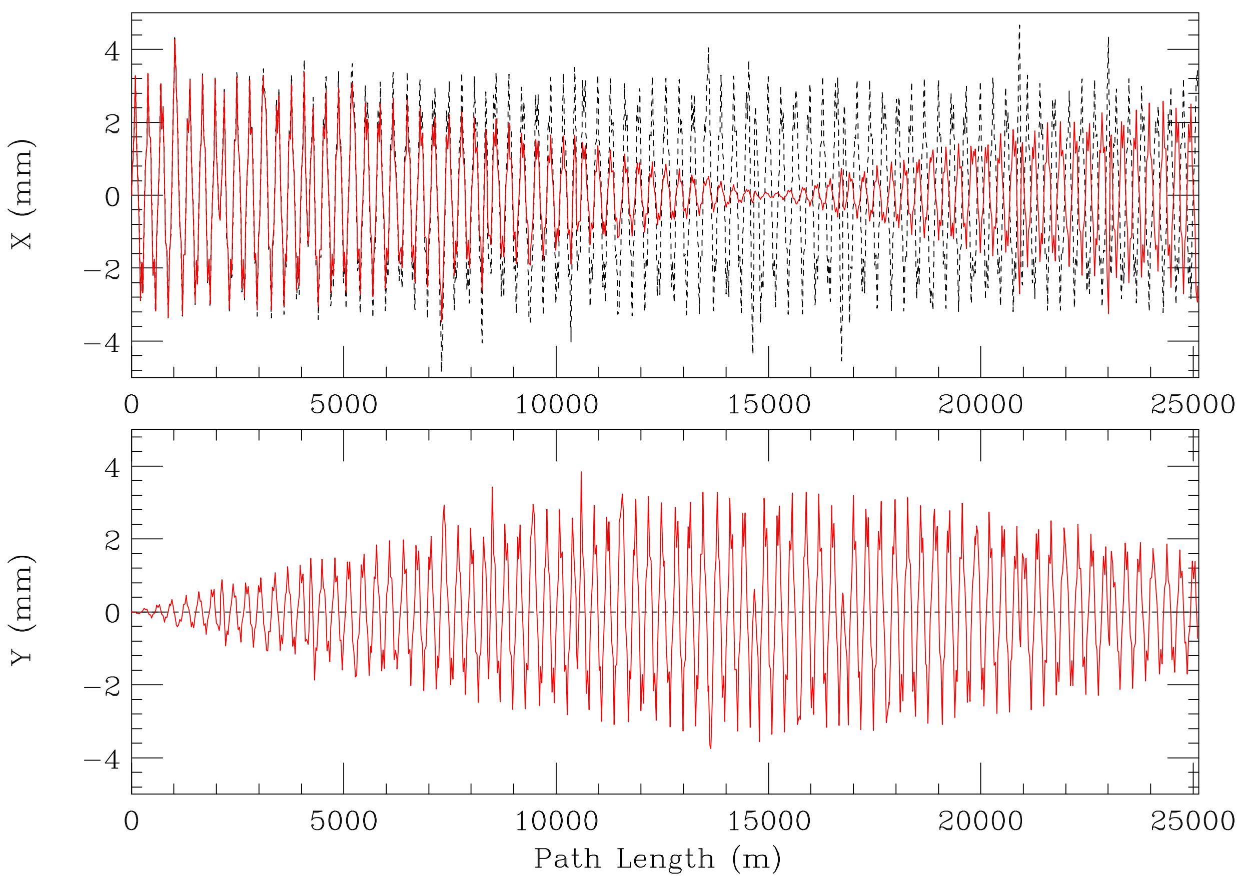


Figure 5: Result of MAD calculation with $a_1 = 1$ unit in each of the Tevatron dipole magnets. The model contains the complete layout of the present Tevatron lattice. Dashed curves are for $a_1 = 0$. (J. Johnstone)

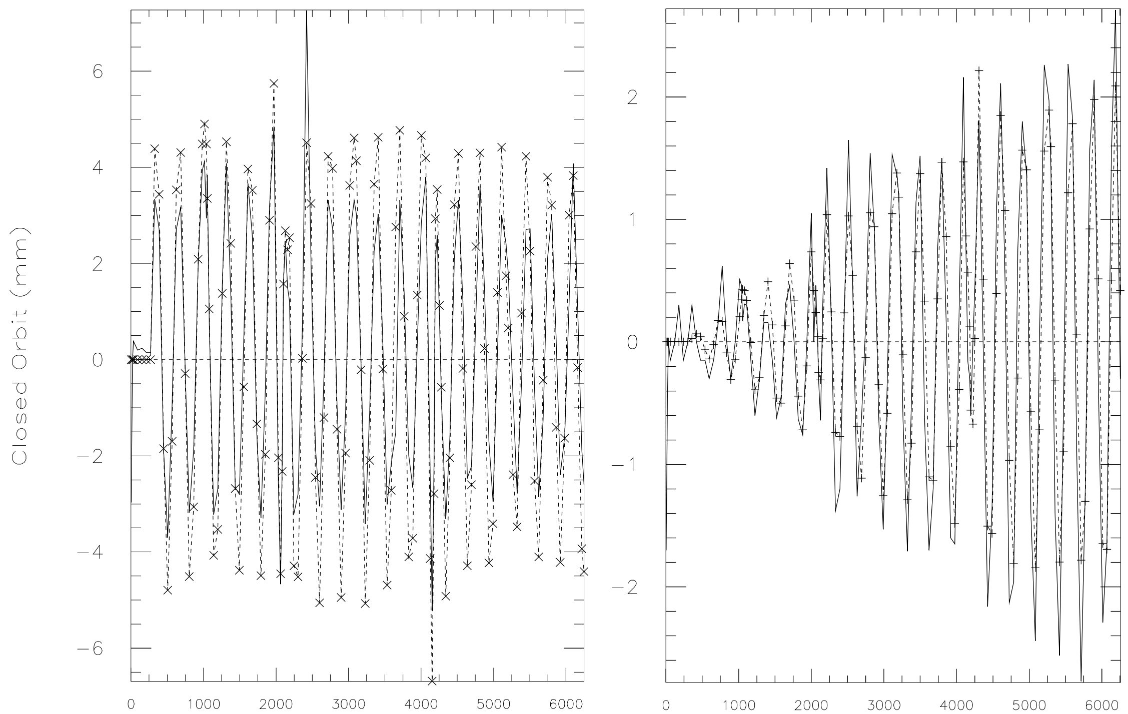


Figure 6: Result of TEVLAT calculation which includes all magnet field error multipoles from measurements made in 1980's. The measured a_1 coefficient for each dipole magnet has 1 unit added to it. The solid lines are from a dead-reckoned calculation, the "plus signs" are data for the horizontal (left) and vertical (right) degrees of freedom. (N. Gelfand)

4 Status of Skew Quadrupole Adjustment Circuits

The discussion of the preceding sections has been concerned with the gross features of transverse coupling. There are a number of coupling sources besides the (yet to be verified) systematic a_1 in the main dipoles, and there are a number of skew quadrupole adjustment circuits. M. Martens[3] has a summary of the situation, and the comments of this section are based on his notes.

In Table 1, we list the skew circuits and their excitations at 150 and 980 GeV. As Martens points out, the excitation ratio of the strong ciruit, T:SQ, between injection and flat-top is significantly different from the energy ratio. The discussion in our present report does not account for this difference.

circuit	elements	150 GeV	980 GeV
T:SQ	42	-2.89 A	-25.98 A
T:SQA0	2	6.29 A	36.55 A
T:SQA4	1	-5.18 A	-33.81 A
T:SQB1	1	0.56 A	3.92 A
T:SQD0	2	0.0 A	0.72 A
T:SQE0	2	0.0 A	0.0 A

Table 1: Skew quadrupole adjustment circuits in the Tevatron. Excitations are given in Amperes.

5 Conclusions and Recommendations

The presence of a systematic a_1 coefficient at the level of 1 unit in the Tevatron dipoles is consistent with skew quadrupole adjustment excitations. Physical measurements on a selection of dipoles support this conclusion. Magnetic measurements are recommended.

With this conclusion, the strong excitation of coupling elements is understood in terms of long-term dipole magnet change. We feel that the sources of both high excitation of steering dipoles and skew quadrupole adjusters have been understood. For the strong steering strengths, refer to the analysis of Syphers[4].

Confirmation of a systematic a_1 term in the dipoles by magnetic measurement is recommended. Given the demonstrated presence of strong coupling terms in the Tevatron, a measurement of horizontal and vertical dispersion in the Tevatron would be useful in the understanding of emittance dilution sources.

References

[1] D. Edwards, "Decoupling of Radial and Vertical Betatron Oscillations at High Energy in the Main Ring," EXP-27, November 8, 1972.

[2] T. Sen, "Coupled motion with skew quadrupoles," March 3, 2003, unpublished note.

[3] M. Martens, "Notes on Quadrupole Fields in the Tevatron," Beams-doc-485, March 2003.

[4] M. Syphers, "Strong Systematic Steering Correction in Regions of the Tevatron," Beams-doc-491, March 2003.